LinAlgBook HW answers

HW I.1

(a) $\begin{bmatrix} 1 & 4 & 3 \\ -1 & 2 & 2 \end{bmatrix}$

(b) NOT DEFINED

(c) $\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 2 \end{bmatrix}$

(f) $\begin{bmatrix} 5 & 10 & 15 & 20 \\ -3 & -6 & -9 & -12 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

HW I. 2 (see Appendix One)

(a) $A^2 = \frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$,

$A^3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $A^4 = \frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$

$A^5 = \frac{1}{2}\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{bmatrix}$, $A^6 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^7 = -A$, $A^8 = -A^2$, $A^9 = -A^3$

$A^{10} = -A^4$, $A^{11} = -A^5$,

$A^{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A^{13} = A$, ...

$A^{n+12} = A^n$, $n = 1, 2, 3, \ldots$

HW I. 2 (b) $\vec{x}_n = A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

= 1st column of A^n, A from (a):

$\vec{x}_1 = \frac{1}{2}\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$, $\vec{x}_2 = \frac{1}{2}\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$,

$\vec{x}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{x}_4 = \frac{1}{2}\begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$

$\vec{x}_5 = \frac{1}{2}\begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix}$, $\vec{x}_6 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

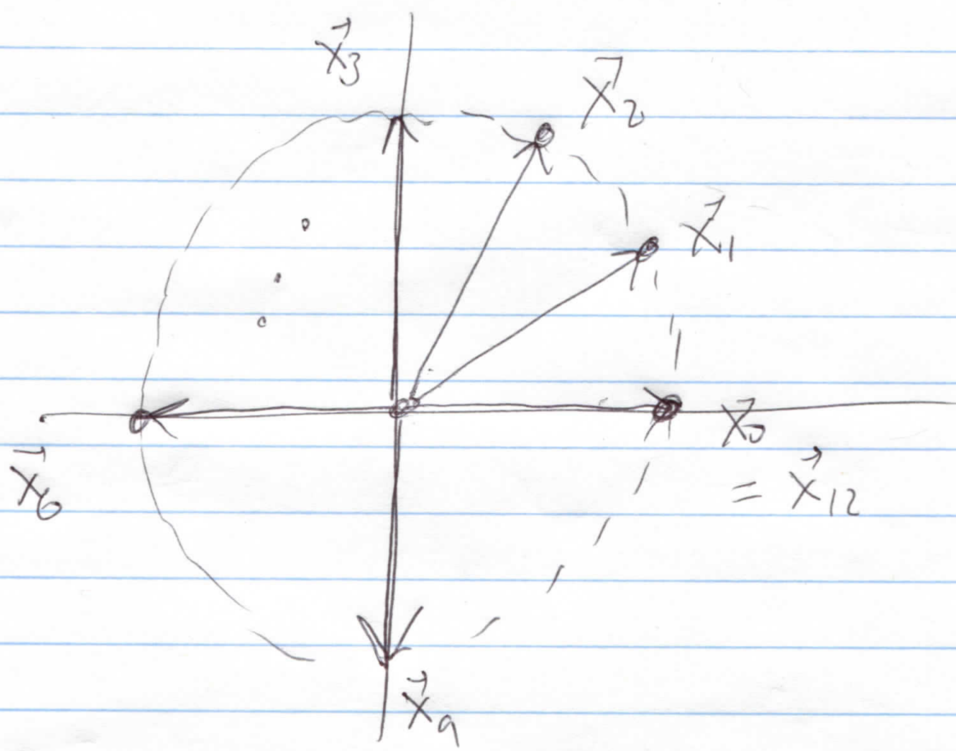
$\vec{x}_7 = -\vec{x}_1$, $\vec{x}_8 = -\vec{x}_2$, $\vec{x}_9 = -\vec{x}_3$,

$\vec{x}_{10} = -\vec{x}_4$, $\vec{x}_{11} = -\vec{x}_5$, $\vec{x}_{12} = \vec{x}_0$

$\vec{x}_{13} = \vec{x}_1$, ...

$$\vec{x}_{n+12} = \vec{x}_n, \quad n = 1, 2, 3, \ldots$$

HW I.2(b) PICTURE



(c) (From (a)) $n = 6$

(d) (From (c)) $n = 12$

HW I.3 This matrix is the square of the matrix in HW I.2.

(a) $A^2 = \frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$,

$A^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $A^4 = -A$, $A^5 = -A^2$,

$A^6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A^7 = A$, ...

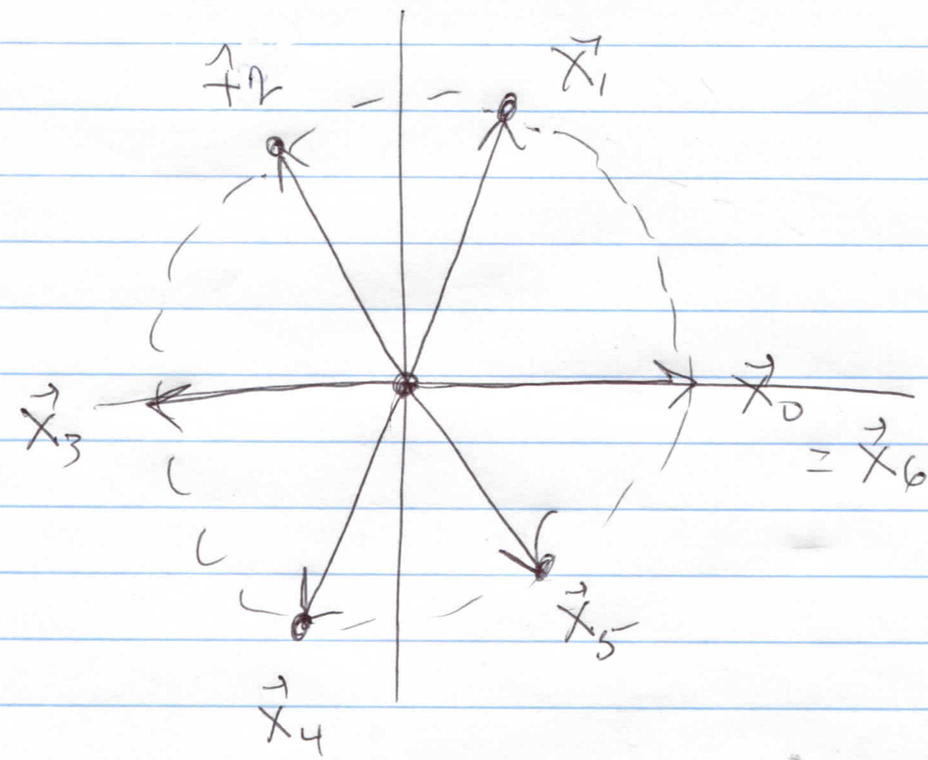
$A^{n+6} = A^n$, $n = 1, 2, 3, \ldots$

(b) $\vec{x}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$, $\vec{x}_2 = \frac{1}{2}\begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$

$\vec{x}_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $\vec{x}_4 = -\vec{x}_1$, $\vec{x}_5 = -\vec{x}_2$,

$\vec{x}_6 = \vec{x}_0$, ... $\vec{x}_{n+6} = \vec{x}_n$, $n = 1, 2, 3, \ldots$

HW I 3 (b) PICTURE



(c) $n = 3$

(d) $n = 6$

HW I.4. 75,014,400 rabbits, 3,840 foxes.

HW I.5 42,300 not on moon, 67,700 on moon.

HW I.6 (3, 0, 4, −7)

HW I.7 (a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^k \end{bmatrix}$

HW I. 8 $AB = \begin{bmatrix} 4 & 6 \\ 3 & 4 \end{bmatrix}$,

$BC = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$ $ABC = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$,

$(AB)^2 = \begin{bmatrix} 34 & 48 \\ 24 & 34 \end{bmatrix}$

HW I. 9 TS_1 is total sales,
TS_2 is total cost,
$(TS_1 - TS_2)$ is profit in a
year; in each matrix, 1^{st}
entry is 1^{st} market, 2^{nd} is
2^{nd} market

HW II.1

(a)
$$\begin{bmatrix} 1 & 0 & 1 & 3 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 2 & 0 & 2 & 6 & 4 \\ 1 & 2 & 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

(b)
$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$+ x_4 \begin{bmatrix} 3 \\ -2 \\ 6 \\ -1 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ -3 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

HW II. 2

(i) $\boxed{x_1 = 1, \quad x_2 = 2, \quad x_3 = -1}$

(ii) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

HW II. 3

(a) $\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -7 \\ 26 \end{bmatrix}}$

HW II, 3 cont!

(b) inconsistent

(c) $x_1 = -x_2 - x_4$

$x_3 = -1 - x_4$

x_2, x_4 arbitrary

HW II. 4

(a) + (c) are consistent;

(b) is not

HW II.5

(a) $x = -\frac{1}{2}, \quad y = \frac{1}{2}$

(b) + (c) inconsistent

(NOTE how small changes, such as (a) → (b) or (a) → (c), change consistent to inconsistent.

(d) $x = (t-s), \quad y = s, \quad z = (1-t), \quad w = t$ (s, t arbitrary)

(e) $x = 1, \quad y = (t-1), \quad z = (1-t), \quad w = t$ (t arbitrary)

(f) $x = 2, \quad y = -2, \quad z = -2, \quad w = 3$

(g) $x = (t-1), \quad y = z = (1-t), \quad w = t$ (t arbitrary)

(h) $x = \frac{1}{2}, \quad y = -\frac{1}{2}, \quad z = -3/2$

(i) inconsistent

HW III. 1

5 − 2 = 3 free variables

HW III. 2

(a) yes, rank of coeff. matrix = 2 (see HW III. 1) = rank of augmented matrix

(b) 5 − 2 = 3 free variables

HW III. 3

(i) Could be no solutions (e.g., "0 = 1" included)

if consistent,

$0 \leq \#$ of free variables ≤ 10

could be unique solution (0 free vars)

could be infinitely many solutions (> 0 free vars)

HW III.3 cont:

(ii) could be no solution,

if consistent,

$5 \le$ # of free variables ≤ 15

can't have unique solution

could have infinitely many solution,

HW III. 4

(i) rank (coeff. matrix) = 2

" (augmented ") = 3

$2 \neq 3 \rightarrow$ inconsistent

(ii) rank (coeff. matrix) = 2

= " (augmented ") = 2

$\rightarrow$ consistent,

$(3-2) = 1$ free variable

HW III. 5

$$\begin{bmatrix} 1 & -4 & 1 \\ 1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

HW III. 6

(i) $x_1 = 3, \quad x_2 = -1, \quad x_3 = -1$

(ii) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(iii) YES, by conclusion of (ii)

HW III.7

$(3-2) = 1$ free variable

HW III.8

(a) $3 \leq$ # of free variables ≤ 10

(b) NO (can't have 0 free vars)

(c) YES (could have > 0 free vars)

HW III. 9

(a) $0 \le \#$ of free variables ≤ 7

(b) YES (can have 0 free vars)

(c) YES (can have > 0 free vars)

HW III. 10

(a) 2 (b) 1

HW III . 11

(a) not invertible

(b) $\frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

HW III . 12

$\begin{bmatrix} 9 & -3/2 & -5 \\ -5 & 1 & 3 \\ -2 & 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix}$

unique : if $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, then

$\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

HW III. 13 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

HW III. 14 (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $B^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if k even,

$B^k = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ if k odd

HW III.15

$$\left(A^k + A^{k+1}\right) = \begin{bmatrix} (-1)^k & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} (-1)^{k+1} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \left((-1)^k + (-1)^{k+1}\right) & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{since } (-1)^n = \begin{cases} 1 & n \text{ even} \\ (-1) & n \text{ odd} \end{cases}$$

HW III.16

$A\begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, so (1, 2, -1, 0) is in $\mathcal{N}(A)$

$A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ is inconsistent, so (0, 1, 2) not in $R(A)$

HW III . 17 (a) 1 (b) 1

(c) 2 (d) 3 (e) 2 (f) 2

(g) 2 (h) 3 (i) 4

HW III . 18 {diagonal matrices}

HW III . 19. $d_1, d_2, \ldots, d_n$ all nonzero

HW III . 20 (a) $\mathcal{R} = \mathbb{R}$, $\mathcal{N} =$

$\{(r-s, r, s, t) \mid r, s, t$ arbitrary$\}$

(b) $\mathcal{R} = \{(b, 0) \mid b$ arbitrary$\}$,

$\mathcal{N}$ same as (a)

(c) $\mathcal{R} = \mathbb{R}^2$, $\mathcal{N} =$

$\{(0, s, s, t) \mid s, t$ arbitrary$\}$

HW III. 20 cont.:

(d) $\mathcal{R} = \mathbb{R}^3$, $\mathcal{N} = \{\vec{0}\}$

(e) $\mathcal{R} = \{(b_1, b_2, b_3, b_4) \mid$
$0 = b_3 - b_1 - 2b_2 = b_4 - b_1 - b_2\}$
$\mathcal{N} = \{(t, -t, t) \mid t$ arbitrary $\}$

(f) $\mathcal{R} = \{(b_1, 0, b_1, b_4) \mid b_1, b_4$ arbitrary$\}$, $\mathcal{N} = \{(0,0)\}$

HW III. 21 $\mathcal{R}(A) = \mathbb{R}^n$,
$\mathcal{N}(A) = \{\vec{0}\}$

HW III. 22 Invertible if & only if $a \neq 1$; inverse is the

$$\frac{1}{(1-a)}\begin{bmatrix} 1 & -1 \\ -a & 1 \end{bmatrix}$$

HW III. 23 rank = 2

$$\mathcal{R}(A) = \left\{ \begin{bmatrix} 0 \\ x_2 \\ x_3 \end{bmatrix} \;\middle|\; x_2, x_3 \text{ arbitrary} \right\}$$

$$\mathcal{N}(A) = \left\{ \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix} \;\middle|\; x_1 \text{ arbitrary} \right\}$$

HW IV. 1

rank(coeff. matrix) = 2

(a) 3 − 2 = 1 free variable

(b) > 0 free variables →
YES, nontrivial solution (of homog.)

(c) rank(augmented matrix)
also = 2, so yes

(d) YES, by (c)

HW IV.2

Anything of form

$$(4-x_3)\begin{bmatrix}-1\\1\\2\\3\end{bmatrix} + (4-3x_3)\begin{bmatrix}1\\2\\0\\1\end{bmatrix}$$

$$+ \; x_3\begin{bmatrix}2\\7\\2\\6\end{bmatrix} \qquad (x_3 \text{ arbitrary})$$

HW IV.3

(a) $\vec{b} = \begin{bmatrix}1\\0\\1\\-1\end{bmatrix} + 2\begin{bmatrix}0\\1\\1\\0\end{bmatrix} - \begin{bmatrix}1\\2\\0\\-1\end{bmatrix}$

(b) not a linear combo

HW IV.4

(a) $\left\{ \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(b) 2

HW IV.5

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

HW IV.6

$\{ (1,0,1,2), (1,2,3,4), (5,6,7,8) \}$

HW IV.7

(a) 3 (b) $4 - 3 = 1$

HW IV.8

ranks different → NO

on both (a) & (b).

HW IV.9

$$\text{rank} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 5 \\ -1 & 3 & 1 \end{bmatrix} \quad (*) = 2$$

(a) NO (rank ≠ # of vectors)

HWIV.9 cont.:

(b) YES (from (a))

(c) YES (from (a) or (b))

(d) YES (square matrix, rank ≠ order)

(e) NO (same reason as (d))

HWIV.10

(a) Impossible, since

$$\text{rank}\begin{bmatrix} x_1 & y_1 & z_1 & w_1 \\ x_2 & y_2 & z_2 & w_2 \\ x_3 & y_3 & z_3 & w_3 \end{bmatrix}$$ cannot equal 4.

HWIV.10 cont. :

(b) possible (necessary,

from (a))

(c) Possible $\left(\left\{ (1,0,0,0),\ (0,0,1,0),\ (0,1,0,0) \right\} \right)$

since rank $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = 3 = \#$ of vector,

(d) Possible $\left(\left\{ (1,0,0,0),\ (2,0,0,0),\ (3,0,0,0) \right\} \right)$

HW IV.11

(a) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

(b) $\{ (1,0,1,-1,0), (0,1,2,0,1) \}$

(c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(d) – (f) SAME as (c)

(g) SAME as (a)

(h) SAME as (c)

HW IV. 12

null space basis: $\left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

basis for range is $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

$(R(A) = \mathbb{R}^2)$

HW IV. 13 $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 9 \\ 0 \\ 1 \end{bmatrix} \right\}$

HW IV. 14

(a) 2

(b) 2

(c) $7 - 2 = 5$

(d) 2

HW IV.15

(a) $3 \le \dim(\mathcal{N}(A)) \le 8$

$0 \le \dim(\mathcal{R}(A)) \le 5$

(b) $0 \le \dim(\mathcal{R}(A)) \le 5$

$0 \le \dim(\mathcal{N}(A)) \le 5$

HW IV.16

(a) ≥ 7 (b) ≤ 7

(c) 7 (d) $6 \le \dim(V) \le 8$

HW IV.17

(a) can't span (too few)

(b) linearly dependent (too many)

HW IV. 18

(a) $\operatorname{rank}\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 3 \\ 0 & 2 & 1 & -3 \\ 1 & 3 & 1 & -2 \end{bmatrix} = \dots 3 \neq 4$, in $\mathbb{R}^4$, 4 vectors

so not a basis

(b) $\operatorname{rank}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{bmatrix} = 4$, in $\mathbb{R}^4$, so (4 vectors)

is a basis

HW IV. 19

(a) $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 5 \\ -1 \end{bmatrix}$

HW IV. 19 cont.:

(b) $x_1 = 1,\ x_2 = 2,\ x_3 = 3$

(c) NO, by (a)

HW IV. 20

NO, since HW III. 6 (i) writes $\vec{0}$ as a nontrivial linear combination of the set of vectors.

HW IV. 21

$$\operatorname{rank}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ -1 & 3 & 1 \end{bmatrix} = \cdots 3 \quad (*)$$

$$\begin{array}{ccc} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{array}$$

HW $\underline{IV}$. 21 cont.:

(a) YES, by $* \left(\text{rank}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = 3 \right)$

(b) NO, by $(*)$ (rank = # of columns)

(c) NO, same reason as (b)

(d) NO, $A\vec{x} = \vec{0} \rightarrow \vec{x} = A^{-1}\vec{0} = \vec{0}$

(see (f))

(e) YES $A\vec{x} = \vec{b} \rightarrow \vec{x} = A^{-1}\vec{b}$

(see (f))

(f) YES, since rank = # of columns, square matrix

HW $\underline{IV}$. 22

(a) possible $\left(\operatorname{rank}[\vec{e}_1\ \vec{e}_2\ \dots\ \vec{e}_6] = 6,\ \text{in } \mathbb{R}^7\right)$

(b) possible $\left(\{\vec{e}_1, 2\vec{e}_1, 3\vec{e}_1, \dots, 6\vec{e}_1\}\right)$

(c) impossible $\operatorname{rank}[\vec{v}_1\ \vec{v}_2\ \dots\ \vec{v}_7] \le 6$, if $\vec{v}_k$ is in $\mathbb{R}^6$, so rank cannot equal 7.

(d) possible (actually necessary, by (c))

HW IV. 23

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ -3 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{(Gauss-Jordan)}$$

$$\{\text{solutions}\} = \left\{ \begin{bmatrix} -2x_3 \\ -x_3 \\ x_3 \\ 0 \end{bmatrix} \middle| \ x_3 \text{ arbitrary} \right\}$$

$\rightarrow$ nontrivial solutions $\rightarrow$

$\boxed{\text{linearly dependent}}$

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HW IV. 24

(a) rank $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 0 & 2 \\ -1 & -5 & 3 \end{array}\right] = \cdots$

rank $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$; rank of coeff. matrix equals rank of augmented matrix

YES

(b) $x_1 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \rightarrow$

(Gauss Jordan) $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$

$\rightarrow$ consistent $\rightarrow$ YES

HW IV. 25

$A \equiv$ matrix in (b), calculate

$\text{rank}(A) = \ldots \; 2 \; (= \text{rank}(A^T))$

(a) 2 (b) $5-2 = 3$

(c) $3-2 = 1$

HW IV. 26

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

(b) $x_1 \begin{bmatrix} 1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

HW IV. 27

$$2\begin{bmatrix}1\\0\\1\\2\end{bmatrix} + \begin{bmatrix}-1\\1\\1\\1\end{bmatrix} + \begin{bmatrix}2\\3\\0\\1\end{bmatrix} = \begin{bmatrix}3\\4\\3\\6\end{bmatrix}$$

HW IV. 28

(a) no (different ranks)

(b) no (same as (a); vector form of linear system)

HW IV. 29

(a) $\operatorname{rank}\begin{bmatrix}1 & 0 & 1\\0 & 2 & 0\\0 & 1 & 1\\-1 & 1 & -1\end{bmatrix} = \ldots 3 = \#$ of vectors

$\rightarrow$ yes

HW IV. 29 cont.:

(b) $$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\rightarrow$ ... (Gauss-Jordan) $x_1 = 0 = x_2 = x_3$

(only trivial soln) $\rightarrow$ yes.

HW IV. 30

(a) yes, by equality of ranks in 1st 2 matrices

(b) no, by nonequality of ranks in 2nd 2 matrices

HW IV.30 cont.

(c) yes by (a) (vector form of linear system)

(d) $5 - 2 = 3$

(e) 2

(f) 2

(g) SAME as (b) no

(h) yes, same as (a)

HW IV.31

(a) $0 \leq \dim(R(A)) \leq 10$

(b) & (c) SAME as (a)

(d) $4 \leq \dim(\mathcal{N}(A)) \leq 14$

HW IV, 32

(a), (b), (c) & (d): between 0 & 10, inclusive

HW IV, 33

(a) can't span ($20 < 32$)

could be lin ind. ($20 \leq 32$)

(b) can't be lin ind ($40 > 32$)

could span ($40 \geq 32$)

(c) max = 32 (dim (V))

min = 1 (nonempty)

(d) max = ∞ min = 32 (dim V)

HW IV. 34

(a) $\dim V \le 13$

(b) $\dim V \ge 17$

(c) $\dim V = 20$

HW IV. 35

(a) 10 (b) 10

(c) $97 - 10 = 87$

HW IV. 36

(a) number of rows ≥ 13

(b) 23 columns

HW IV. 37

Since $V = \mathcal{N}([1 \quad 1 \quad 1 \quad 1])$,

$\dim(V) = 4 - \operatorname{rank}[1 \quad 1 \quad 1 \quad 1] = 3$

(a) $\operatorname{rank}\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & -3 & 2 \\ 2 & 0 & -1 & -1 \end{bmatrix} = \cdots 3$,

so yes, yes, & yes, to (i), (ii) & (iii).

(b) $\operatorname{rank}\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & -3 & 2 \end{bmatrix} = 2$

= # of vectors, so Lin ind

only 2 vectors, so doesn't span V & not a basis

(since $2 < \dim V$)

HW IV. 37 cont.

(c) $\text{rank}\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & -7 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix} = \ldots 3 = \dim(V)$,

so span V

4 vectors, $\dim(V) = 3 \rightarrow$
not lin ind & not a basis

(d) $\text{rank}\begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix} = \ldots 2$

$< 3 = \dim(V)$, & 3 vectors, so

no, no, & no, to (i), (ii) & (iii)

HW IV. 38

3 vectors in $\mathbb{R}^3$, $\dim(\mathbb{R}^3) = 3$,

so look at

$$\operatorname{rank}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 4 \\ 3 & 1 & 7 \end{bmatrix} = \ldots 2 \neq 3,$$

so $\boxed{\text{not a basis}}$

HW IV. 39

3 = # of vectors in basis

HW IV. 40

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

1

HW V.1 (a) (-2) (b) (-10)

(c) 7 (d) (-5)

HW V.2 $\left| \det \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} \right| = 2$

HW V.3 det $= 15 \neq 0$, so yes

HW V.4 $\det(2A) = 2^{10}(\det A) = 3 \cdot (2^{10})$

$= 3{,}072$

$\det(A^7) = (\det A)^7 = 3^7 = 2{,}187$

HW V.5 $(\det A)(\det B) = \det(AB) =$

$\det(I) = 1 \rightarrow$ det A & det B

are nonzero $\rightarrow$ A & B are invertible

HW1.5 cont.

$AB = I \rightarrow$

$B = A^{-1}AB = A^{-1}I = A^{-1}$

&

$A = ABB^{-1} = IB^{-1} = B^{-1}$

HW VI. 1

$$-\frac{7}{6}(1,1,0,0,-2) + 3(1,1,1,1) - \frac{1}{4}(0,0,2,-2,0)$$

HW VI. 2

$$-(0,1,0,1) + (1,1,0,-1) = (1,0,0,-2)$$

HW VI. 3

$$\frac{12}{2}\vec{w}_1 + \left(-\frac{3}{2}\right)\vec{w}_2 + \frac{5}{4}\vec{w}_3 + \frac{1}{4}\vec{w}_4$$

HW VI. 4

$y = \frac{7}{10} + \frac{8}{25}x$; error

is $\frac{\sqrt{20}}{10} \simeq 0.447$

HW VI. 5

$$y = 3 - \frac{18}{5}x + x^2$$

HW VI. 6

$$x = \frac{2}{3} + z$$

$$y = -\frac{1}{3} - z$$

z arbitrary

HW VI. 7

$$\begin{bmatrix} 5 & 2 & 30 & 44 \\ 2 & 30 & 44 & 354 \\ 30 & 44 & 354 & 812 \\ 44 & 354 & 812 & 4{,}890 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ -30 \\ -86 \\ -456 \end{bmatrix}$$

HW VI. 8

(a) $y = 3$

(b) $y = 3 + b(x-2)$ (b arbitrary)

(c) $y = \frac{3}{2}x$

(d) $y = 3 + b(x-2) + c(x^2-4)$
(b, c arbitrary)

All graphs go thru $(2,3)$

HW VI. 9

(a), (b), (c) & (d) all have
error $3\sqrt{2}$

HW VI. 10

$$\frac{1}{3}(1, 2, 0, -2, 0)$$

HW VI. 11

$$\frac{1}{2}(1,1,0,0) - \frac{5}{47}(1,-1,3,6) + \frac{1}{5}(0,0,2,-1)$$

HW VI. 12

$$\frac{2}{3}(1,1,-1) - \frac{1}{6}(1,1,2) + \frac{1}{2}(1,-1,0)$$

HW VI. 13

NO, $\|(0,-1,1)\| = \sqrt{2} \neq 1$

HW VI. 14

$(0,1,-1) \times (1,0,-3) = (-3,-1,-1)$

(or $(3,1,1)$)

HW VI. 15

$\frac{1}{2} \| (0,3,0) \times (1,0,1) \| = \frac{\sqrt{18}}{2}$

HW VI. 16

$\| (0,1,2) \times (0,3,2) \| = 4$

HW VI. 17

$\| (1,0,5) \times (2,1,0) \| = \sqrt{126}$

HW VI.18

$\alpha_1 = 2$

HW VI.19

$\vec{v}_1 = \text{proj}_{\vec{b}}(\vec{a}) = \dots \frac{32}{77}(4,5,6) = \vec{v}_1$

$\vec{v}_2 = (1,2,3) - \frac{32}{77}(4,5,6)$

HW VI.20

$\vec{v}_1 = (0,1,1,0), \quad \vec{v}_2 = \frac{1}{2}(-2,1,-1,2)$

$\vec{v}_3 = (2,1,0,0) - \left[\frac{1}{2}(0,1,1,0) - \frac{3}{10}(-2,1,-1,2)\right]$

HW VI.21

$y = \frac{5}{11}x$

HW VI. 22

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 10 & 0 & 34 \\ 0 & 0 & 34 & 0 \\ 0 & 34 & 0 & 130 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 7 \\ 9 \end{bmatrix}$$

HW VI. 23

$$x^* = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix};$$

error is $\sqrt{2}/2$

HW VI. 24 (a) For $1 \le j, i \le n$ the $(ij)^{th}$ entry of A^TA is (the i^{th} row of A^T) $\cdot$ (j^{th} column of A)

HWVI. 24 (a) cont.:

$= (i^{\text{th}}$ column of $A) \cdot (j^{\text{th}}$ column of $A)$

$$= \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

since the columns of A are orthonormal.

This implies that $A^T A = I$;

HW V.5 implies that A is invertible, with $A^{-1} = A^T$.

(b) $A\vec{x} \cdot A\vec{y} = \vec{x} \cdot (A^T A)\vec{y}$

$= \vec{x} \cdot \vec{y}$.

(c) $\|A\vec{x}\|^2 = A\vec{x} \cdot A\vec{x} = \vec{x} \cdot \vec{x} = \|\vec{x}\|^2$ &

$0 = \vec{x} \cdot \vec{y} = A\vec{x} \cdot A\vec{y} \rightarrow A\vec{x} \perp A\vec{y}$

HW VI. 25: By Theorem 6.63, we need numbers a, b such that $\begin{bmatrix} a \\ b \end{bmatrix}$ is the least squares solution of

$$(*) \quad A \begin{bmatrix} a \\ b \end{bmatrix} = \vec{b},$$

where $A \equiv \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$, $\vec{b} \equiv \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

We leave it to the reader to calculate

$$A^T A = \begin{bmatrix} n & \left(\sum_{k=1}^{n} x_k\right) \\ \left(\sum_{k=1}^{n} x_k\right) & \left(\sum_{k=1}^{n} x_k^2\right) \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} \sum_{k=1}^{n} y_k \\ \sum_{k=1}^{n} x_k y_k \end{bmatrix}$$

so that the Normal Equation for $(\ast)$ are

$$\begin{bmatrix} n & \left(\sum_{k=1}^{n} x_k\right) \\ \left(\sum_{k=1}^{n} x_k\right) & \left(\sum_{k=1}^{n} x_k^2\right) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} y_k \\ \sum_{k=1}^{n} x_k y_k \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & \overline{x} \\ \overline{x} & \overline{(x^2)} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \overline{y} \\ \overline{(xy)} \end{bmatrix};$$

solving the latter linear system tells us that the least-squares approximating line is $y = a + bx$, where

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$$b = \frac{\left(\overline{(xy)} - \bar{x}\bar{y}\right)}{\left(\overline{(x^2)} - (\bar{x})^2\right)},$$

$$a = \bar{y} - \bar{x}b.$$

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HW VII. 1

(a) $\begin{bmatrix} 3 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 1 & -19 \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

(b) $\frac{1}{10}\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$

(c) $\frac{1}{10}\begin{bmatrix} -8 & 6 \\ 6 & 8 \end{bmatrix}$

(d) $\frac{1}{20}\begin{bmatrix} (1+3\sqrt{3}) & (-\sqrt{3}+3) \\ (3+9\sqrt{3}) & (-3\sqrt{3}+9) \end{bmatrix}$

(e) $\frac{1}{20}\begin{bmatrix} (1-3\sqrt{3}) & (3-9\sqrt{3}) \\ (\sqrt{3}+3) & (3\sqrt{3}+9) \end{bmatrix}$

HW VII. 2

(a) $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 0 & 0 \\ 0 & 1 & -19 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$

(b) $\frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(c) $\frac{1}{10}\begin{bmatrix} (\sqrt{3}+2) & (-1+2\sqrt{3}) \\ (2\sqrt{3}+4) & (-2+4\sqrt{3}) \end{bmatrix}$

(d) $\frac{1}{10}\begin{bmatrix} (\sqrt{3}-2) & (2\sqrt{3}-4) \\ (1+2\sqrt{3}) & (2+4\sqrt{3}) \end{bmatrix}$

(e) $\frac{1}{5}\begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$

HW VII. 3 $F_1 = 1 = F_2$, $F_3 = 2$,
$F_4 = 3$, $F_5 = 5$, $F_6 = 8$,
$F_7 = 13$, $F_8 = 21$, $F_9 = 34$, $F_{10} = 55$

HW VII. 4 $A = \frac{1}{12}\begin{bmatrix} 7 & 2 & 6 \\ 4 & 8 & 0 \\ 1 & 2 & 6 \end{bmatrix}$

HW VII. 5 $A = \frac{1}{12}\begin{bmatrix} 6 & 1 & 3 \\ 6 & 9 & 3 \\ 0 & 2 & 6 \end{bmatrix}$

HW VII. 6. $A = \frac{1}{6}\begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

HW VII. 7 $A = \frac{1}{28}\begin{bmatrix} 7 & 0 & 0 \\ 18 & 24 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

HW VIII . 1

$\{(-t, -t, t) \mid t$ arbitrary

HW VIII . 2

$\{(s-2t, s, t) \mid s, t$ arbitrary$\}$

HW VIII . 3

$\boxed{\text{eigenvalue} = -2}$

HW VIII . 4

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

HW VIII . 5

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & (-1) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$P = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \rightarrow A = PDP^{-1}$$

HW VIII. 6

$$A^k = \begin{bmatrix} 1 & 0 & 0 \\ \left(1-\left(\frac{1}{2}\right)^k\right) & \left(\frac{1}{2}\right)^k & 0 \\ -\left(\frac{1}{2}\right)^k & \left(\frac{1}{2}\right)^k & 0 \end{bmatrix}$$

HW VIII. 7

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & (-3) \end{bmatrix}, \quad P = \begin{bmatrix} (-1) & 1 & 1 \\ 0 & 1 & (-4) \\ 1 & 1 & 1 \end{bmatrix}$$

$\rightarrow A = PDP^{-1}$

HW VIII. 8

(a) $P = \begin{bmatrix} 1 & 1 \\ (-1) & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

$\rightarrow A = PDP^{-1}$

HW VIII. 8 cont.

(b) $P \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad D \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\rightarrow \quad A = PDP^{-1}$

(c) $P \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 \\ 0 & 4 & 1 & 0 & (-1) \\ 0 & 5 & 0 & 1 & 2 \end{bmatrix},$

$D \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow$

$A = PDP^{-1}$

HW VIII. 9

$2 \pm \sqrt{11}$

HW VIII. 10

$(A^k - A)$ = zero matrix, $k = 1, 2, 3, \ldots$

HW VIII. 11

(a) (3)

(b) (1)

(c) (1)

(d) (3)

(e) (1)

(f) (2)

HW $\underline{\text{VIII}}$. 12

$$\frac{1}{2}\begin{bmatrix} (-1)^{k+1} & (-1)^k & (-1)^k \\ \left(2+2(-1)^{k+1}\right) & 2(-1)^k & \left(-2+2(-1)^k\right) \\ \left(-2+(-1)^{k+1}\right) & (-1)^k & \left(2+(-1)^k\right) \end{bmatrix}$$

HW $\underline{\text{VIII}}$. 13

(a) $\begin{bmatrix} 3-(2^{k+1}) \\ -3+3(2^k) \end{bmatrix} = \vec{x}_k, \quad k=1,2,3,\ldots$

(b) $\vec{x}_{10} = \begin{bmatrix} 3-(2^{11}) \\ -3+3(2^{10}) \end{bmatrix} = \begin{bmatrix} -2045 \\ 3069 \end{bmatrix}$

HW $\underline{\text{VIII}}$. 14

$$A^k = \begin{bmatrix} 2^{k+1} & -(2^k) \\ 2^{k+1} & -(2^k) \end{bmatrix}$$

HW $\underline{VIII}$. 15

(a) $$P = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & -1 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ $A = PDP^{-1}$

HW VIII.15 cont.

(b) not diagonalizable

(geom. mult. of 1) = 2 $\neq$

3 = (alg. mult. of 1)

(c) $$P = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 1 & 0 & 3 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} (-1) & 0 & 0 & 0 & 0 \\ 0 & (-1) & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \rightarrow A = PDP^{-1}$$

HW VIII. 15 cont.

(d) not diagonalizable

sum of dimension of eigenspaces is 4, but A is (5×5)

HW VIII. 16

5 & (-2)

HW VIII. 17

$$E_3 = \left\{ \begin{bmatrix} s \\ t \\ t \end{bmatrix} \;\middle|\; s, t \text{ arbitrary} \right\}$$

HW VIII. 18

3

HW VIII. 19

$$P = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$\rightarrow \quad A = PDP^{-1}$

HW VIII. 20

$$P = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & (-2) & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\rightarrow \quad A = PDP^{-1}$

HW VIII. 21

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -4 \\ -1 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$$

$A = PDP^{-1}$

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HW VIII. 22

$$P = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & (-1) & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$\rightarrow A = PDP^{-1}$

HW ~~VIII~~. 23

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\rightarrow A = PDP^{-1}$

#24 (a) two simple eigenvalues: 2, 3.
(b) -1 is an eigenvalue of multiplicity two.
(c) 7 is a simple eigenvalue; 3 is an eigenvalue of multiplicity two.
(d) -1 is an eigenvalue; 1 is an eigenvalue of multiplicity two.
(e) 3 is a simple eigenvalue; 1 is an eigenvalue of multiplicity two.
(f) three simple eigenvalues: 0, 1, 3.

#25. (a) for 2: $\{(s,s) \mid s \text{ is arbitrary }\}$; for 3: $\{(2s,s) \mid s \text{ is arbitrary }\}$.
(b) for -1: $\{(s,s) \mid s \text{ is arbitrary }\}$.
(c) for 7: $\{(s,0,s) \mid s \text{ is arbitrary }\}$; for 3: $\{(-s-\frac{1}{2}t, t, s) \mid s, t \text{ are arbitrary }\}$.
(d) for 1: $\{(0,0,s) \mid s \text{ is arbitrary }\}$; for -1: $\{(-s,s,0) \mid s \text{ is arbitrary }\}$.
(e) for 1: $\{(-(s+t), s, t) \mid s, t \text{ are arbitrary }\}$; for 3: $\{(t,0,t) \mid t \text{ is arbitrary }\}$.
(f) for 0: $\{(-s,s,s) \mid s \text{ is arbitrary }\}$; for 1: $\{(0,-s,s) \mid s \text{ is arbitrary }\}$;
for 3: $\{(2s,s,s) \mid s \text{ is arbitrary }\}$.

#26. 7 is a simple eigenvalue, and -2 is an eigenvalue of multiplicity two.

#27. b and d are not diagonalizable.
(a). Let

$$S \equiv \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \quad D \equiv \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix},$$

then the matrix equals SDS^{-1}.
(c). Let

$$S \equiv \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}, \quad D \equiv \begin{bmatrix} 7 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

then the matrix equals SDS^{-1}.
(e). Let

$$S \equiv \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad D \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

then the matrix equals SDS^{-1}.
(f). Let

$$S \equiv \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad D \equiv \begin{bmatrix} 0 & 0 & \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

then the matrix equals SDS^{-1}.

#28 $A \equiv \frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ -6 & 4 & -2 \\ -3 & 1 & 1 \end{bmatrix}$. #29 $S \equiv \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$. #30 $\frac{1}{2}\begin{bmatrix} (5+\sqrt{33}) & 0 \\ 0 & (5-\sqrt{33}) \end{bmatrix}$.

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#31. (a)

$$\frac{1}{2}\begin{bmatrix}(1+(-1)^k) & (1+(-1)^{k+1})\\(1-(-1)^k) & (1-(-1)^k)\end{bmatrix}.$$

(b)

$$\frac{1}{4}\begin{bmatrix}(3^k+3(-1)^k) & (3^k+(-1)^{k+1})\\(3^{k+1}+3(-1)^{k+1}) & (3^{k+1}+(-1)^k)\end{bmatrix}.$$

(c)

$$\frac{1}{4}\begin{bmatrix}2(7^k+3^k) & (7^k-3^k) & 2(7^k-3^k)\\0 & 4\cdot 3^k & 0\\2(7^k-3^k) & (7^k-3^k) & 2(7^k+3^k)\end{bmatrix}.$$

(d)

$$\frac{1}{2}\begin{bmatrix}(1+3^k) & (3^k-1) & (3^k-1)\\0 & 2 & 0\\(3^k-1) & (3^k-1) & (3^k+1)\end{bmatrix}.$$

HW VIII.32

$$A^k=\frac{1}{2}\begin{bmatrix}(-1+(\frac{1}{3})^{k-1}) & (1-(\frac{1}{3})^k) & (-1+(\frac{1}{3})^k)\\(-2-(\frac{1}{2})^{k-2}+6(\frac{1}{3})^k) & (2+(\frac{1}{2})^{k-1}-2(\frac{1}{3})^k) & (-2+2(\frac{1}{3})^k)\\(1-(\frac{1}{2})^{k-2}+(\frac{1}{3})^{k-1}) & (-1+(\frac{1}{2})^{k-1}-(\frac{1}{3})^k) & (1+(\frac{1}{3})^k)\end{bmatrix}$$

#33. $$A^k=\frac{1}{2}\begin{bmatrix}2^{k+1} & 0 & 0\\(2^k-3^k) & (2^k+3^k) & (3^k-2^k)\\(2^k-3^k) & (3^k-2^k) & (2^k+3^k)\end{bmatrix}.$$

for any k.

HW VIII. 34 $F_{42} = 267,914,296$

$F_{29} = 514,229$